Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 24: The Natural Exponential Function

Definition 1. **Principal:** *The original, or initial, amount of money borrowed is the* ***principal****, denoted by* P.

Definition 2. **Interest:** *A fee charged for borrowing a lender's money is called the* ***interest*** *and denoted by* I.

Definition 3. **Interest Rate:** *The* ***interest rate*** *is the percent charged for the use of the principal for given period of time. The interest rate which is denoted by* r *is expressed as a decimal. Unless stated otherwise the period is assumed to be one year.*

Definition 4. **Simple Interest:** *The amount of interest computed only on the principal is called the* ***simple interest****.*

Definition 5. **Simple Interest Formula:** *The simple interest* I *on a principal* P *at a rate* r *per year for* t *years is given by*

$$I = Prt.$$

Example 1. *Juanita has deposited* \$8000 *in a bank for five years at a simple interest rate of* 6%.

a. *How much interest she will receive?*

b. *How much money will be in her account at the end of five years?*

Solution:

a. $P = \$8000,\ r = 0.06,\ t = 5.$

$$\begin{aligned} I &= Prt \\ &= (8000)(0.06)(5) \\ &= \$2400. \end{aligned}$$

b. *In five years, the amount* A *she will receive is the original principal plus the interest earned:*

$$\begin{aligned} A &= P + I \\ &= 8000 + 2400 \\ &= \$10,400. \end{aligned}$$

Definition 6. **Compound Interest:** *The interest paid on both the principal and the accrued (previously earned) interest is called the* ***compound interest****.*

Definition 7. **Compound Interest Formula:** *The formula is*

$$A = P(1 + \frac{r}{n})^{nt},$$

where

1. *A = amount after t years. It is also called the **future value**.*
2. *P = principal.*
3. *r = annual interest rate (expressed as a decimal number).*
4. *n = number of times the interest is compounded each year.*
5. *t = number of years.*

Example 2. *Assuming that $100 is deposited in a bank that pays 5% annual interest. Find the future value A after one year if the interest is compounded:*

a. *Semiannually.*

b. *Quarterly.*

Solution: *In the following computations, $P = 100$, $r = 0.05$, and $t = 1$.*

a. *Since the interest is compounded semiannually, then $n = 2$ and*

$$\begin{aligned} A &= P(1 + \frac{r}{n})^{nt} \\ &= (100)(1 + \frac{0.05}{2})^{(2)(1)} \\ &\approx \$105.06. \end{aligned}$$

b. *Since the interest is compounded quarterly, then $n = 4$ and*

$$\begin{aligned} A &= P(1 + \frac{r}{n})^{nt} \\ &= (100)(1 + \frac{0.05}{4})^{(4)(1)} \\ &\approx \$105.09. \end{aligned}$$

Definition 8. **Euler Number:** *The **Euler number**, to 15 places, is*

$$e = 2.718281828459045.$$

Definition 9. **Continuous Compound Interest Formula:** *When interest is compounded **continuously**, then the future value is given by the formula:*

$$A = Pe^{rt},$$

where

1. *A = amount after t years.*
2. *P = principal.*

3. $r =$ *annual interest rate (expressed as a decimal number).*

4. $t =$ *number of years.*

Example 3. *Find the amount when a principal of* \$8300 *is invested at a* 7.5% *annual rate of interest compounded continuously for eight years and three months.*

Solution: *We convert eight years and three months to* $t = 8.25$ *years,* $P = 8300$ *and* $r = 0.075$. *Then*

$$\begin{aligned} A &= Pe^{rt} \\ &= (8300)e^{(0.075)(8.25)} \\ &\approx \$15,409.83. \end{aligned}$$

Definition 10. **The Natural Exponential Function:** *The* ***natural exponential function*** *is given by*

$$f(x) = e^x,$$

with ***base*** e. *The graph of* $f(x)$ *is*